Solutions to JEE Main Home Practice Test - 5 | JEE - 2024

PHYSICS

SECTION-1

1.(D) $w_f = \dfrac{1}{\sqrt{LC}}$ frequency will not change

$Q = \dfrac{\omega L}{R}\left(R\uparrow Q\downarrow\right)$; $\quad Q = \dfrac{\omega L}{R} = \dfrac{\omega}{\Delta B}$

$\Delta B = \dfrac{R}{L}\left(R\uparrow \Delta B\downarrow\right)$

So, the bandwidth of the resonance circuit will decrease.

2.(B) In figure 2, 4 and 5. P-crystals are more positive as compared to N-crystals.

3.(B) B_W is magnetic field of wire.

$B_1 = B_0 + B_W$; $\quad B_2 = B_0 - B_W$

$\Rightarrow \quad B_0 = \dfrac{B_1 + B_2}{2}$

4.(C) $V = V_0 \sin(\omega t)$

$\omega = 2\pi f = 100\pi$

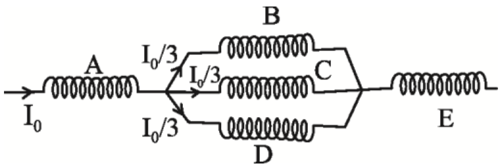
(i) $\quad 440\sqrt{2} = 440\sqrt{2}\sin(100\pi t_1)$

$\sin(100\pi t_1) = 1$; $\quad t_1 = \dfrac{1}{200}$ sec

(ii) $\quad 440 = 440\sqrt{2}\sin(100\pi t_2)$

$\sin(100\pi t_2) = \dfrac{1}{\sqrt{2}} \Rightarrow t_2 = \dfrac{1}{400}$

5.(C)



$B \propto i$

So, field at centre of C $= \dfrac{3}{3} = 1T$

6.(C) $f = 200\,MHz$

$\vec{E} = \vec{B}\times\vec{V}$

$\vec{E} = 3\times10^{-8}k\times3\times10^{8}i$

$\vec{E} = (9k\times i) = 9\,jV/m$

7.(C) $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$; $\quad \dfrac{\lambda_p}{\lambda_c} = \dfrac{\frac{h}{m_p v_p}}{\frac{h}{m_e v_e}} = \dfrac{m_e v_e}{m_p v_p}$

$\frac{1}{2} = \frac{m_e}{m_p} \times \frac{v_e}{16v_e}$; $\quad m_p = \frac{m_e}{8}$

i.e., $\frac{1}{8}$ times the mass of electron

8.(D) $C_V = \frac{fR}{2}$

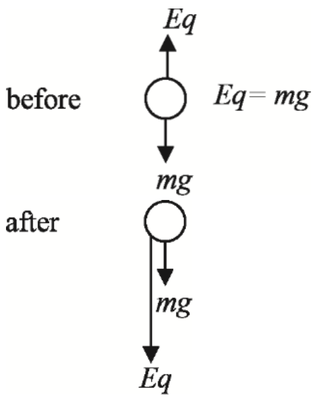
f is = 3 for monoatomic but f is 5 for diatomic at normal temperature. But $f > 5$ for diatomic gases.
At high temperature energy of vibration also increases taken that into account C_V increases since molecules of monoatomic gas do not vibrate, its C_V remains same.

9.(C) If distant objects are blurry then the problem is Myopia.
If objects are distorted then the problem is Astigmatism.

10.(B) $l_1 = \frac{\lambda}{4}, l_2 = \frac{3}{4}\lambda \Rightarrow \lambda = 2(l_2 - l_1)$, $v = n\lambda$ = 330 m/s,

also $\frac{dv}{v} = \frac{dl_2 + dl_1}{l_2 - l_1}$, dv = 1.98 m/s.

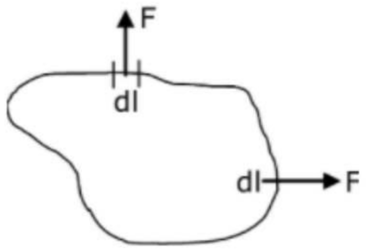
11.(D)



12.(C) $T^2 \propto R^3$; $\quad \frac{T_1}{T_2} = \frac{R_1^{3/2}}{R_2}$

$\frac{T_1}{T_2} = \left(\frac{R}{4R}\right)^{3/2}$; $\quad \frac{T}{T_2} = \frac{1}{8}$; $\quad T_2 = 2T$

13.(D) Force on each small wire segment be along radially outward and equal so, it will take the shape of a circle and parallel to the field.



14.(A) $D = 1m; d = 1.0\,mm, \lambda = 5890\text{Å}$

Distance between first and third bright fringe $= 2\beta = 2\lambda D / d \quad = 1178 \times 10^{-6} m$

15.(C) ΔF = (Patm)A = 101300

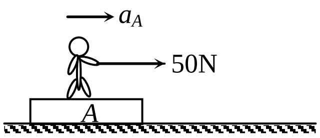
16.(D) $P = \vec{F}.\vec{v} = ma \times at = ma^2 t \quad [as\, u = 0]$

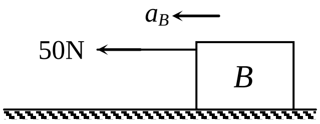
$$= m\left(\frac{v_1}{t_1}\right)^2 t = \frac{mv_1^2 t}{t_1^2} \qquad \left[As\, a = \frac{v_1}{t_1}\right]$$

17.(A) Maximum friction force between block A and boy = $0.5 \times 80 \times 10 = 400\,\text{N} > 50\,\text{N}$

So, $a_A = \frac{50}{200} = \frac{1}{4}\,\text{m/s}^2$

$a_B = \frac{50}{100} = \frac{1}{2}\,\text{m/s}^2$





So relative a = 0.75 m/s^2

Relative velocity

$v = 0 + 0.75 \times 4 = 3$ m/s

Friction between boy and block A

$$f = 120 \times \frac{1}{4} = 30\ \text{N}$$

18.(B) $E \propto \frac{1}{r}$

$r \propto \frac{1}{m}$

$E \propto m$

Ionization potential $= 13.6\left(mass_\mu\right)\left(mass_e\right)eV \;\equiv 13.6 \times n\,eV$

19.(B) Work done for the adiabatic process is:

$$W_{CD} = \frac{nR\Delta T}{1-\gamma} = \frac{p_f v_f - p_i v_i}{1-\gamma} = \frac{200 \times 4 - 6 \times 100}{1-1.4}$$

$$W_{CD} = \frac{800-600}{-0.4} = \frac{-200}{0.4} = -500J$$

$$W_{CD} = -500J$$

20.(D) $v_H = u\cos\theta = 6$

$$v_v = \sqrt{v^2 - u^2\cos^2\theta} = 8$$

$$t_1 = \frac{u\sin\theta - 8}{10}$$

$$t_2 = \frac{u\sin\theta + 8}{10}$$

$$t_2 - t_1 = \frac{8 \times 2}{10} = 1.6\ \text{s}$$

SECTION – 2

1.(10) Take horizontal surface as reference level for gravitational potential energy

By every conservation

$$K_i + U_i = K_f + U_f$$

$$v_B^2 = 500/5 = 100$$

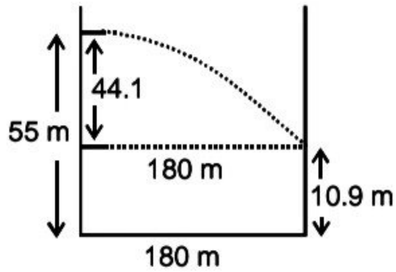
$$v_B = 10\,m/s; \quad x = 10$$

2.(6) impulse J = mV.

angular impulse $= J\dfrac{\ell}{2} = I\omega$

So $mv\dfrac{\ell}{2} = I\omega$ or $\omega = \dfrac{mv\ell}{2I} = \dfrac{mv\ell}{2\left(\dfrac{m\ell^2}{12}\right)} = \dfrac{6v}{\ell} = 6\,\text{rad/s}$

3.(60) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 44.1}{9.8}}$

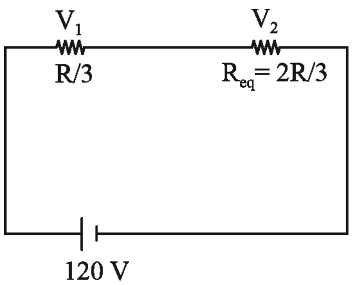


$t = 3\ sec$

R = uT

$\dfrac{180}{3} = u\ ;\ u = 60\ m/s$

4.(80)



$$\frac{v_1}{v_2} = \frac{R/3}{2R/3} = \frac{1}{2}$$

$$v_2 = \frac{2}{3}\times 120 = 80V$$

5.(48) $F_A = F_B; \qquad F_B = 4r_A$

$\Delta l_a = 2\,mm\ ; \quad \Delta l_B = 4\,mm$

$Y = stress/strain = \dfrac{F/A}{\dfrac{\Delta l}{l}}$

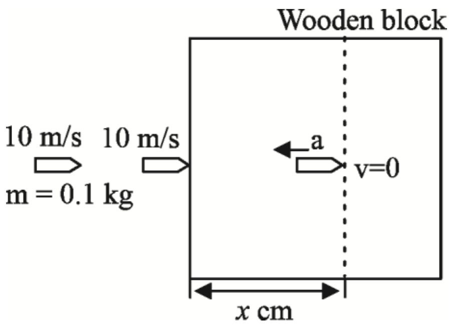
$\Delta l / l = F / AY$

$I = AY\Delta l / F$

$$\frac{l_a}{l_b}=\frac{\pi r_a^2\times Y\times \Delta l_a\times F}{\pi r_b^2\times Y\times \Delta l_b\times F}$$

$$\frac{l_a}{l_b}=\frac{r_a^2}{\left(4r_a\right)^2}\times\frac{2n}{6}=\frac{1}{16\times 3}=\frac{1}{48}$$

6.(50)



$$F=10N=ma\Rightarrow a=\frac{10}{0.1}=-100\,m/s^2$$

$$v_f=0, v_i=10\,m/s$$

$$v_f^2-v_i^2=2as\Rightarrow 0^2-10^2=2\times\left(-100\right)\times s$$

7.(10) $P_i=P_f$

$$m\times 10\hat{i}+0=\left(m\times 0\right)+\left\{m\times 10\,j+m\vec{v}\right\}$$

$$\vec{v}=10\hat{i}-10\,j$$

$$\left|\vec{v}\right|=10\sqrt{2}$$

8.(20) $kq^2\left(\frac{1}{r}-\frac{1}{2r}\right)=\frac{1}{2}m(V^2)$

$$\frac{kq^2}{2r}=\frac{1}{2}mV^2$$

$$\frac{kq^2}{r}=\frac{1}{2}m(\sqrt{2}V)^2=\frac{1}{2}m(V_{\max}^2)$$

$$V_{\max}=V\sqrt{2}=(10\sqrt{2})\sqrt{2}=20\text{m/s}$$

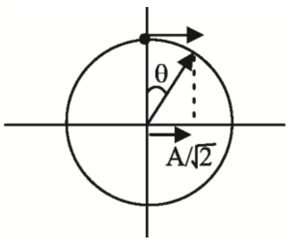
9.(12) v_1 = 5m/s

Let the bird flies at an angle θ with east-west direction

$13\sin\theta=5$

Speed with respect to earth $=13\cos\theta=12$

10.(4)



$$T=2\,\text{sec},\ \theta=45^\circ$$

$$\omega=\frac{360^\circ}{T}$$

$$t=\frac{\theta}{\omega}=\frac{45\times T}{360}=\frac{T}{8}=\frac{1}{4}$$

CHEMISTRY

SECTION - 1

1.(A) Ionisation potential of nitrogen is more than that of oxygen. This is because nitrogen has more stable half-filled p-subshell. $(N = 1s^2 2s^2 2p^3, O = 1s^2 2s^2 2p^4)$

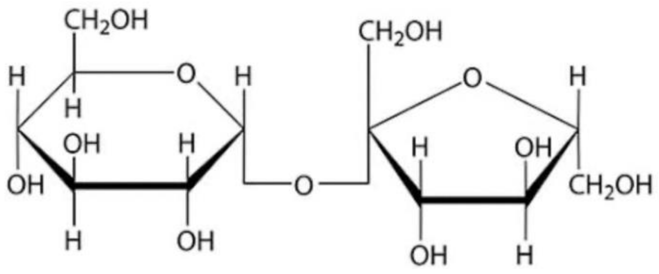
2.(A) $N_2H_4 / glycol / KOH \rightarrow$ Wolff-Kishner reduction

Zn-Hg/conc. HCl $\rightarrow$ Clemmensen reduction

$OsO_4 \rightarrow$ Alkenes to vicinal diols

$HIO_4 \rightarrow$ Vicinal glycol to carbonyl compound

3.(B) Sucrose is a non-reducing sugar because it has no free carbonyl group.

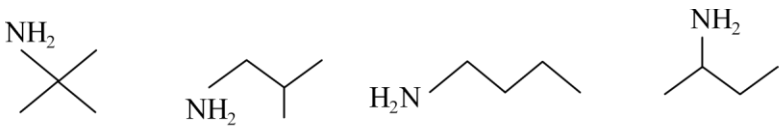


4.(B) Two groups or atoms attached to doubly bonded carbon atoms should be different for G.I.

$CH_3 - CH_2 - CH = CH_2$

5.(D) Because N lone pair are directly involved in conjugation.

6.(A)



7.(A)

8.(A) $S^{2-} + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5(NOS)]$

9.(B) (B) Option does not have P.O.S. : (A) and (C) have P.O.S.

10.(A)

11.(C) Correct order of acidic strength is :

$HClO < HClO_2 < HClO_3 < HClO_4$

12.(D) $l = 2$

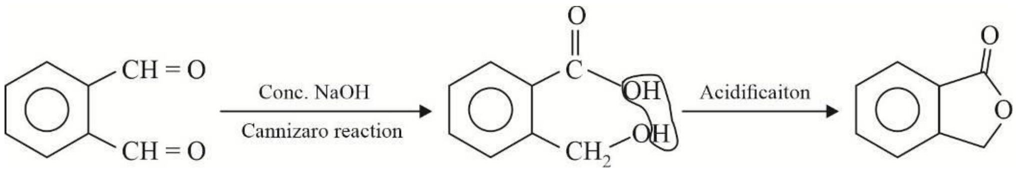
$Fe(26) = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$

$Fe^{2+}(24) = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$

$l = 2 \Rightarrow d$

Electron present in (l = 2) d-orbitals = 6

13.(C)



14.(D)

$$\underset{H_2}{n_1 = \frac{100 \times 200}{RT}} \qquad \underset{He}{n_2 = \frac{200 \times 100}{RT}}$$

$$P(V_1 + V_2) = (n_1 + n_2)RT$$

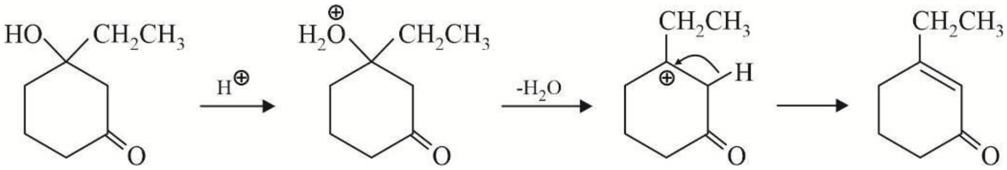
$$P \times (300\,\text{mL}) = 2\left(\frac{100 \times 200}{RT}\right) \times RT$$

$$P \times 300\ \text{mL} = 400 \times 100$$

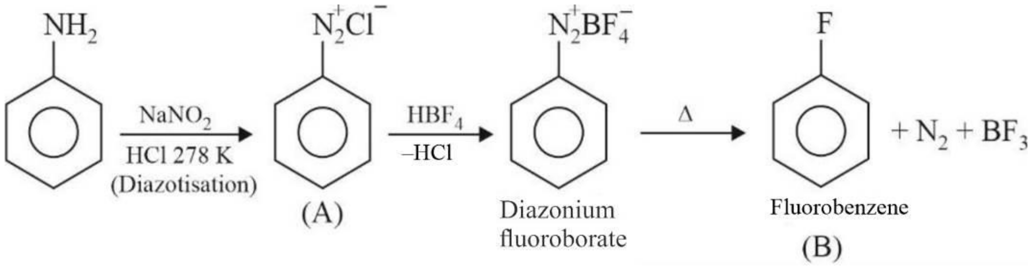
$$P = \frac{400 \times 100}{300} = 133.33\ \text{torr}$$

15.(B) Cs is most electropositive element.

16.(A)

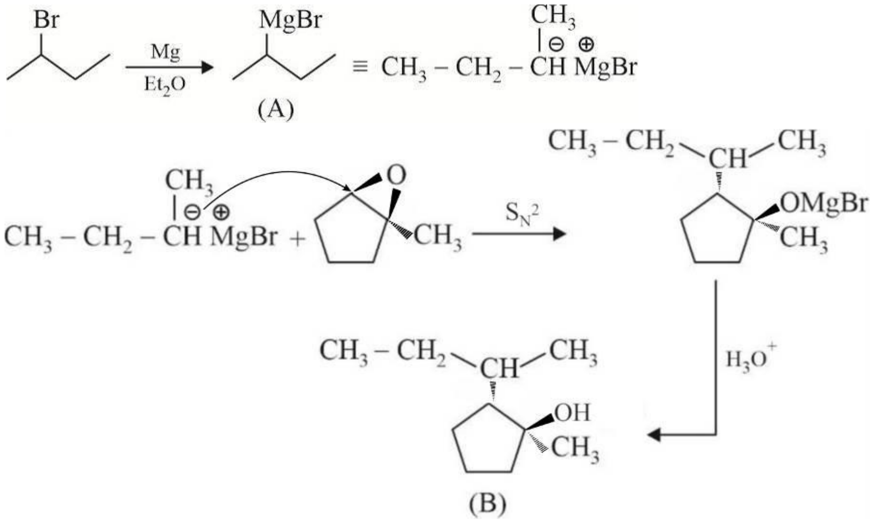


17.(C)



The reaction of diazonium chloride with fluoroboric acid (HBF_4) is called Balz-Schiemann reaction.

18.(A)



19.(C) Partial hydrolysis of cellulose gives the disaccharide cellobiose $(C_{12}H_{22}O_{11})$. Cellobiose resembles maltose (which on acid catalysed hydrolysis yields two molar equivalents of D-glucose) in every respect except one : the configuration of its glycosidic linkage.

20.(C)

SECTION - 2

1.(62) $\alpha = \dfrac{1-i}{1-\dfrac{1}{n}} = \left(\dfrac{1-i}{1-n}\right)n$

$0.8 = \dfrac{1-i}{1-\dfrac{1}{n}}$; $i = 0.4$

$\Delta T = iK_f \times m$

$0.3 = 0.4 \times 1.86 \times \dfrac{w_A \times 1000}{m_A \times w_{H_2O}}$

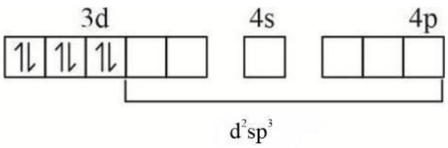
$0.3 = 0.4 \times 1.86 \times \dfrac{2.5 \times 1000}{m_A \times 100}$

$m_A = 62$

2.(0) Metal ion having electron configuration from $d^4 \,\&\, d^7$ can form both low spin and high spin complexes.

$[NiF_6]^{-2}$

$Ni^{+4} = [Ar]4s^\circ 3d^6$



The number of unpaired electron = 0
In this compound, pairing occurs.

3.(4)

Species	Bond order
N_2^+	2.5
N_2^-	2.5
O_2	2
O_2^+	2.5
F_2	1
B_2	1
C_2^+	1.5
CN^-	3
NO^+	3

4.(3) $C_3H_8O_3 + CH_3MgI \longrightarrow CH_4$

$\frac{0.092}{92}$ $\qquad\qquad$ $\frac{67}{22400}$

= 0.001 mole $\qquad$ = 0.003 mole

Therefore, 3 active hydrogen atoms are present.

5.(10) $4C_3H_6 + 6NO \rightarrow 4C_3H_3N + 6H_2O + N_2$

Moles of propylene taken $= \frac{420 \times 10^3}{42} = 10^4$ moles

Moles of C_3H_3N produced $= 10^4$ moles

Weight of $C_3H_3N = 10^4 \times 53\,g = 530\,kg$

$\therefore$ wt of $C_3H_3N = 530kg = x$; $\quad \frac{x}{53} = \frac{530}{53} = 10$

6.(1) In first and second sets of experiments, [B] are same whereas [A] is changing:

Let us suppose the rate law is $R = K[A]^m [B]^n$

$$R_1 = K[0.01]^m [0.01]^n = 0.005$$

$$R_2 = K[0.02]^m [0.01]^n = 0.01$$

$$\frac{R_1}{R_2} = \frac{1}{2} = \left(\frac{0.01}{0.02}\right)^m = \left(\frac{1}{2}\right)^m \Rightarrow m = 1$$

In first and third sets of experiment [A] are same, [B] is changing, therefore

$$R_1 = K[0.01]^m [0.01]^n = 0.005$$

$$R_3 = K[0.01]^m [0.02]^n = 0.005$$

$$\frac{R_1}{R_3} = 1 = \left(\frac{0.01}{0.02}\right)^n = \left(\frac{1}{2}\right)^n \Rightarrow n = 0$$

Reaction is of first order with respect to A and of zero order with respect to B

The overall order is 1.

7.(46) $2Fe^{3+} + 2e^- \longrightarrow 2Fe^{2+}$ & $2I^- \longrightarrow I_2 + 2e^-$

Hence, for the given cell reaction, n = 2

$$\Delta_r G^\circ = -nFE^0_{cell} = -2 \times 96500 \times 0.236\,J = -45.55\,kJ\,mol^{-1}$$

8.(42) $\Delta_r H = \sum(BDE)_R - \sum(BDE)_P$

$$\Delta_r H = [4\Delta H_{C-H} + 4\Delta H_{Cl-Cl} - 4\Delta H_{C-Cl} - 4\Delta H_{H-Cl}]$$

$$= [4 \times 414 + 4 \times 243 - 4 \times 331 - 4 \times 431]$$

$\Delta_r H = -420\,kJ$; $\quad \frac{-\Delta_r H}{10} = +\frac{420}{10} = 42$

9.(4) $pH = pK_a + \log\frac{0.06}{0.05}$; $\quad pH = 3.744 + \log 1.2 = 3.823$

10.(6) $C_4H_x + \left(4 + \frac{x}{4}\right)O_2 \rightarrow 4CO_2 + \frac{x}{2}H_2O$

$10\,ml \quad \left(4 + \frac{x}{4}\right) \times 10$; $\quad \left(4 + \frac{x}{4}\right) \times 10 = 55$

$4 + \frac{x}{4} = 5.5$; $\quad \frac{x}{4} = 1.5$; $\quad x = 6$

MATHEMATICS

SECTION-1

1.(B) $f(x)=\begin{cases}-1 & -2\le x<0\\ x-1 & 0\le x\le 2\end{cases}$

Since, $x\in[-2,2]$. Therefore $|x|\in[0,2]$.

Consequently

$f(|x|)=|x|-1$ for all $x\in[-2,2]$

$$\Rightarrow \quad f(|x|)=\begin{cases}-x-1 & \text{for all } x\in[-2,0]\\ x-1 & \text{for all } x\in[0,2]\end{cases} \qquad \ldots (1)$$

Now, $f(x)=\begin{cases}-1 & -2\le x<0\\ x-1 & 0\le x\le 2\end{cases}$

$$\Rightarrow \quad |f(x)|=\begin{cases}1 & -2\le x<0\\ 1-x & 0\le x<1\\ x-1 & 1\le x\le 2\end{cases} \qquad \ldots (2)$$

From (1) and (2) we get

$$g(x)=f(|x|)+|f(x)|=\begin{cases}-x-1+1 & -2\le x<0\\ x-1+1-x & 0\le x<1\\ x-1+x-1 & 1\le x\le 2\end{cases}$$

$$\therefore \quad g(x)=\begin{cases}-x & -2\le x<0\\ 0 & 0\le x<1\\ 2(x-1) & 1\le x\le 2\end{cases}$$

2.(A) The system has non-zero solution if

$$\Delta=\begin{vmatrix}\sin 3\theta & -1 & 1\\ \cos 2\theta & 4 & 3\\ 2 & 7 & 7\end{vmatrix}=0$$

Applying $R_2\to R_2+4R_1$ and $R_3\to R_3+7R$

$$\Rightarrow \quad \Delta=\begin{vmatrix}\sin 3\theta & -1 & 1\\ \cos 2\theta+4\sin 3\theta & 0 & 7\\ 2+7\sin 3\theta & 0 & 14\end{vmatrix}=0 \quad \Rightarrow \quad 14\cos 2\theta+56\sin 3\theta-14-49\sin 3\theta=0$$

$$\Rightarrow \quad 2\cos 2\theta+\sin 3\theta-2=0 \quad \Rightarrow \quad 2(\cos 2\theta-1)+\sin 3\theta=0$$

$$\Rightarrow \quad -4\sin^2\theta+3\sin\theta-4\sin^3\theta=0 \quad \Rightarrow \quad -\sin\theta\left(4\sin^2\theta+4\sin\theta-3\right)=0$$

$$\Rightarrow \quad \sin\theta(2\sin\theta-1)(2\sin\theta+3)=0 \quad \Rightarrow \quad \sin\theta=0, \sin\theta=\frac{1}{2}, \sin\theta=\frac{-3}{2}$$

$$\Rightarrow \quad \theta=n\pi, \theta=n\pi+(-1)^n\frac{\pi}{6}$$

3.(A) Since, $\alpha\in z^n+2z^{n-1}+3z^{n-2}-18z+12=0 \quad \Rightarrow \quad \alpha^n+2\alpha^{n-1}+3\alpha^{n-2}-18\alpha+12=0$

$\Rightarrow \quad \left(\alpha^n - 1\right) + 2\left(\alpha^{n-1} - 1\right) + 3\left(\alpha^{n-3} - 1\right) = 18\left(\alpha - 1\right)$

$\Rightarrow \quad \left(\dfrac{\alpha^n - 1}{\alpha - 1}\right) + 2\left(\dfrac{\alpha^{n-1} - 1}{\alpha - 1}\right) + 3\left(\dfrac{\alpha^{n-3} - 1}{\alpha - 1}\right) = 18$

$\Rightarrow \quad \left(1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1}\right) + 2\left(1 + \alpha + \alpha^2 + \ldots + \alpha^{n-2}\right) + 3\left(1 + \alpha + \alpha^2 + \ldots + \alpha^{n-3}\right) = 18$

$\Rightarrow \quad \left(1 + |\alpha| + |\alpha|^2 + \ldots + |\alpha|^{n-1}\right) + 2\left(1 + |\alpha| + |\alpha|^2 + \ldots + |\alpha|^{n-2}\right)$

$+3\left(1 + |\alpha| + |\alpha|^2 + \ldots + |\alpha|^{n-3}\right) \geq 18$

As $n \to \infty$ and $|\alpha| < 1$

$\Rightarrow \quad \dfrac{1}{1 - |\alpha|} + \dfrac{2}{1 - |\alpha|} + \dfrac{3}{1 - |\alpha|} > 18 \qquad \Rightarrow \quad \dfrac{1}{1 - |\alpha|} > 3$

$\Rightarrow \quad 1 - |\alpha| < \dfrac{1}{3} \qquad \Rightarrow \quad |\alpha| > \dfrac{2}{3}$ and $|\alpha| < 1$

4.(C) Here, $t_1 = a_1 = 1 - \left(1 - a_1\right)$

And $t_2 = \left(1 - a_1\right)a_2 = 1 - a_1 - \left(1 - a_1\right)\left(1 - a_2\right)$

Also, $t_3 = \left(1 - a_1\right)\left(1 - a_2\right)a_3$

$\Rightarrow \quad t_3 = \left(1 - a_1\right)\left(1 - a_2\right) - \left(1 - a_1\right)\left(1 - a_2\right)\left(1 - a_3\right)$

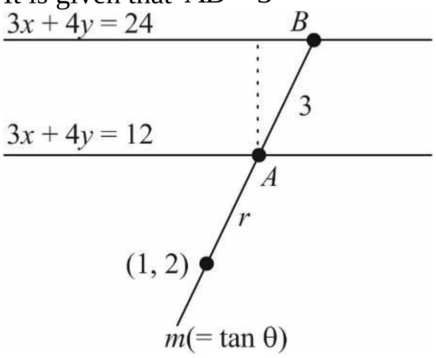
$\therefore \quad S = \sum_{r=1}^{n} t_n = 1 - (1 - a_1)(1 - a_2)\ldots(1 - a_n) \qquad \Rightarrow \quad S = 1 - \prod_{i=1}^{n}(1 - a_1)$

5.(D) Let the required line be $y - 2 = m\left(x - 1\right)$

This line meets $3x + 4y - 12 = 0$ and $3x + 4y - 24 = 0$ at

$A\left(\dfrac{4 + 4m}{3 + 4m}, \dfrac{6 + 9m}{3 + 4m}\right)$ and $B\left(\dfrac{16 + 4m}{3 + 4m}, \dfrac{6 + 21m}{3 + 4m}\right)$ respectively.

It is given that $AB = 3$



$\Rightarrow \quad \left(\dfrac{12}{3 + 4m}\right)^2 + \left(\dfrac{12}{3 + 4m}\right)^2 m^2 = 9 \qquad \Rightarrow \quad m = \dfrac{7}{24}$

So, the required line is $y - 2 = \dfrac{7}{24}(x - 1)$

Or $7x - 24y + 41 = 0$

6.(C) Since, $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix} \quad \Rightarrow \quad \Delta = 1([z]+1+[y])+1(+[x]) = [z]+[y]+[x]+1$$

$\therefore \quad \max[\Delta] = 2+0+1+1 = 4$

7.(C) Given that $f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases} \Rightarrow f(x) = \begin{cases} (x+1)e^{0} & x < 0 \\ 0 & x = 0 \\ (x+1)e^{-\frac{2}{x}} & x > 0 \end{cases}$

According to definition of $|x|$

Now, $f(0^-) = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$\Rightarrow \quad f(0^-) = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) \quad \Rightarrow \quad f(0^-) = \lim_{h \to 0} (0-h+1) = 1$

Also, $f(0^+) = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) \quad \Rightarrow \quad f(0^+) = \lim_{h \to 0} (0+h+1)e^{-\frac{2}{h}} = 0$

Also, $f(0) = 0$

Since, $f(0^-) \neq f(0^+) = f(0)$

$\therefore \quad f(x)$ is not continuous at $x = 0$. $\quad \therefore \quad f(x)$ is not differentiable at $x = 0$.

$f(-2) = -1$; $\quad f(x) \in [-1,1)$ for $x \in [-2,0)$

$f(x) = 0$ for $x = 0$; $\quad f(2) = \frac{3}{e}$; $\quad f(x) \in \left(0, \frac{3}{e}\right]$ for $x \in (0,2]$

Hence, (C) is correct.

8.(B) $\hat{c} = m\hat{a} + n\hat{b} + p(\hat{a} \times \hat{b})$

Taking Dot product with $\hat{a}$ & $\hat{b}$ resp., we get

$m = n = \cos\theta$

Also, $|\vec{c}| = |\cos\theta\hat{a} + \cos\theta\hat{b} + p(\hat{a} \times \hat{b})| = 1$

$\cos^2\theta + \cos^2\theta + p^2 = 1$ (Squaring both sides)

$\Rightarrow \quad \cos\theta = \pm\frac{\sqrt{1-p^2}}{\sqrt{2}} \quad \Rightarrow \quad -\frac{1}{\sqrt{2}} \leq \cos\theta \leq \frac{1}{\sqrt{2}} \quad \{\text{for } \theta \in R\} \quad \therefore \quad \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

9.(A) The value of the number xyz is 100x + 10y + z, where as x and z can be any two of the digits 1 to 9. But y can only be any one of the digits 1, 4, 9.

$\therefore$ Total value of the 3 digit numbers so formed is

$V = 100(1+2+3+...+9).9.3 + 10(1+4+9).9.9 + (1+2+...+9).9.3$

$\therefore \quad V = 121500 + 11340 + 1215 = 134055$

10.(A) Since $x^2+y^2+4y-5=0$

Centre is $C_1(0,-2)$ and Radius $r_1=\sqrt{4+5}=3$

Let $C_2(h,k)$ be the centre of the smaller circle and its radius r_2.

Since, $C_1C_2=4$

$\Rightarrow \quad \sqrt{h^2+(k+2)^2}=3+r_2=4 \quad \Rightarrow \quad r_2=1$

But $k=r_2=1$

Hence, using equation (1), we have

$4=\sqrt{h^2+(1+2)^2} \quad \Rightarrow \quad 16=h^2+9 \quad \therefore \quad h=\pm\sqrt{7}$

Since, $h>0$

Hence, required circle is $\left(x-\sqrt{7}\right)^2+(y-1)^2=1$

11.(D) Since, $h'(x)=[f(x)]^2+[g(x)]^2 \quad \Rightarrow \quad h''(x)=2f(x)f'(x)+2g(x)g'(x)$

$\Rightarrow \quad h''(x)=2f(x)g(x)+2g(x)f''(x) \quad \Rightarrow \quad h''(x)=2f(x)g(x)-2g(x)f(x)=0$

$\therefore \quad h'(x)$ is a constant function, such that

$h'(x)=k \quad \Rightarrow \quad h(x)=kx+\lambda$

Since, $h(0)=2$ and $h(1)=4 \quad \therefore \quad \lambda=2$ and $4=k+2$

Hence, $h(x)=2(x+1)=y$

12.(A) $\sin^{-1}\left(\dfrac{1+t^2}{2t}\right)$ is defined if $\left|\dfrac{1+t^2}{2t}\right|\le 1$ {by definition}

$\Rightarrow \quad \dfrac{1+t^2}{2|t|}\le 1 \quad \Rightarrow \quad 1+|t|^2\le 2|t|$

$\Rightarrow \quad (1-|t|)^2\le 0 \quad \Rightarrow \quad (1-|t|)^2=0$

$\Rightarrow \quad |t|=1 \quad$ or $\quad t=\pm 1$

$\therefore \quad k=\sin^{-1}1=\dfrac{\pi}{2} \quad \{\because\ k>0\}$

13.(D) $\dfrac{e^{x^2}\left(2x+x^3\right)}{\left(3+x^2\right)^2}=\dfrac{1}{2}e^{x^2}\left(\dfrac{1}{3+x^2}-\dfrac{1}{\left(3+x^2\right)^2}\right)2x$

Hence, $\displaystyle\int\frac{e^{x^2}\left(2x+x^3\right)}{\left(3+x^2\right)}dx=\frac{1}{2}\frac{e^{x^2}}{\left(3+x^2\right)}+c$

14.(C) The equation of a straight line which is at a unit distance from the origin is

$x\cos\theta+y\sin\theta=1$ …(1)

Differentiating w.r.t. x

$\cos\alpha+\dfrac{dy}{dx}\sin\alpha=0$ …(2)

On eliminating α from (1) and (2), we get

$$\sin\alpha\left(y-x\frac{dy}{dx}\right)=1$$

$$\Rightarrow \quad \left(y-x\frac{dy}{dx}\right)=\operatorname{cosec}\alpha \qquad \ldots(3)$$

Also, slope = $\dfrac{dy}{dx}=-\cot\alpha$ $\qquad$ {using (2)}

$$\operatorname{cosec}\,\alpha=\sqrt{1-\cot^2\alpha}=\sqrt{1-\left(\frac{dy}{dx}\right)^2} \qquad \ldots(4)$$

From (3) and (4), we have $\left(y-\dfrac{xdy}{dx}\right)^2=\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}$

15.(B) $f(x)=\dfrac{\left[\sin^{-1}\left(\dfrac{x}{1000}\right)+\cos^{-1}\left(\dfrac{x}{1000}\right)\right]x!}{\sqrt{1000x^2-999\{x\}^2-900[x]^2-50x+60}}$

$$\Rightarrow \quad f(x)=\frac{\pi x!}{\sqrt[2]{1000x^2-999\{x\}^2-990[x]^2-50x+60}}$$

Since, $x!$ is there, the function $f(x)$ is defined for only non-negative integral values i.e. $x\in I^{+}+\{0\}$

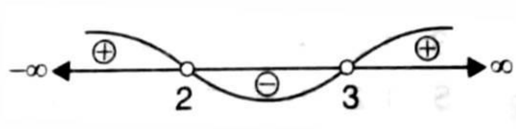
Hereafter, $\{x\}=0 \quad \because \quad x\in I^{+}$ and $[x]=x$

$$\Rightarrow \quad f(x)=\frac{\pi x!}{\sqrt[2]{1000x^2-990x^2-50x+60}} \qquad \Rightarrow \quad f(x)=\frac{\pi x!}{\sqrt[2]{10x^2-50x+60}}$$

Which is defined if $10x^2-50x+60>0$

$$\Rightarrow \quad (x-2)(x-3)>0$$

$$\Rightarrow \quad x\in(-\infty,2)\cup(3,\infty)$$



Since, x is any non-negative integer, hence the correct answer is:

$x\in I^{+}+\{0\}-\{2,3\}$

16.(B) Let $E=\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$

$$\Rightarrow \quad E=\lim_{x\to 0}\frac{-2\sin\left(\dfrac{\sin x+x}{2}\right)\sin\left(\dfrac{\sin x-x}{2}\right)}{x^4}$$

$$\Rightarrow \quad E=-2\left\{\lim_{x\to 0}\frac{\sin\left(\dfrac{\sin x+x}{2}\right)}{x^2}\right\}\left\{\lim_{x\to 0}\frac{\sin\left(\dfrac{\sin x-x}{2}\right)}{x^2}\right\}$$

$$\Rightarrow \quad E=-2\left\{\lim_{x\to 0}\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\frac{\left(\frac{\sin x+x}{2}\right)}{x^2}\right\}\left\{\lim_{x\to 0}\frac{\sin\left(\frac{\sin x-x}{2}\right)}{\left(\frac{\sin x-x}{2}\right)}\frac{\left(\frac{\sin x-x}{2}\right)}{x^2}\right\}$$

$$\Rightarrow \quad E=-2\left\{1.\lim_{x\to 0}\frac{\sin x+x}{2x^2}\right\}\left(1.\lim_{x\to 0}\frac{\sin x-x}{2x^2}\right)$$

$$\Rightarrow \quad E=-\frac{2}{4}\left\{\lim_{x\to 0}\frac{x\left(\frac{\sin x}{x}+1\right)}{x^2}\right\}\left\{\lim_{x\to 0}\frac{\sin x-x}{x^2}\right\}$$

$$\Rightarrow \quad E=-\frac{1}{2}\left\{\lim_{x\to 0}\frac{2}{x}\right\}\left\{\lim_{x\to 0}\frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-...-x}{x^2}\right\}$$

$$\Rightarrow \quad E=-\lim_{x\to 0}\left(-\frac{x^3}{3!}+\frac{x^5}{5!}-...\right)$$

$$\Rightarrow \quad E=-\lim_{x\to 0}\frac{x^3\left(-\frac{1}{6}+\frac{x^2}{120}-...\right)}{x^3} \qquad \{\because x\to 0 \,\&\, x\neq 0\} \qquad \therefore \qquad E=-\left(-\frac{1}{6}\right)=\frac{1}{6}$$

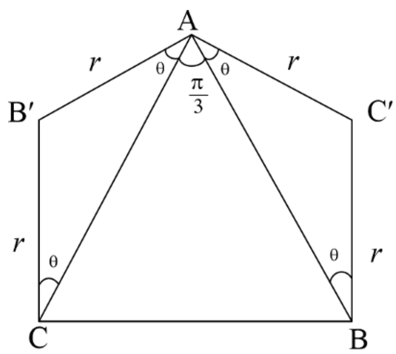
17.(D) In isosceles $\Delta A B' C'$,

$$\sin\left(\frac{\pi}{6}+\theta\right)=\frac{(B'C')/2}{r}$$

$$\Rightarrow \quad B'C' = 2r\left(\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right)$$

From $\Delta A B' C$, $\cos\theta=\frac{1}{r}$.

$$\Rightarrow \quad B'C' = 1+\sqrt{3(r^2-1)}.$$



18.(A) Let $E=(a+b+c)^n+(a-(b+c))^n$

$$\Rightarrow \quad E=2\left\{{}^nC_0a^n+{}^nC_2a^{n-2}(b+c)^2+{}^nC_4a^{n-4}(b+c)^4+....\left(\frac{n+1}{2}\right)\text{terms}\right\}$$

The number of terms in the expansion will be $\left(\frac{n+1}{2}\right)$ terms $\because$ n is odd.

Hence, the number of Distinct terms are: $1+(2+1)+(4+1)+....\left(\frac{n+1}{2}\right)$ terms i.e. $\left(\frac{n+1}{2}\right)^2$

19.(A) Let $S=E(\theta)E(\phi)$

$$\Rightarrow \quad S=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$$

$\Rightarrow \quad S=\begin{bmatrix}\cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi)\\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi)\end{bmatrix}$

Since, θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$

$\Rightarrow \quad S=\begin{bmatrix}\cos\theta\cos\phi\cos\left(2n+1\right)\frac{\pi}{2} & \cos\theta\sin\phi\cos\left(2n+1\right)\frac{\pi}{2}\\ \cos\phi\sin\theta\cos\left(2n+1\right)\frac{\pi}{2} & \sin\theta\sin\phi\cos\left(2n+1\right)\frac{\pi}{2}\end{bmatrix} \quad \therefore \quad S=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

20.(B) $OM_r = OA_r + \frac{OA_{r+1}-OA_r}{2}$

$=\frac{OA_r+OA_{r+1}}{2}$

$=\frac{1}{2}\left\{OA_1\times k^{r-1}+OA_1\times k^r\right\}$

$=\frac{OA_1}{2}(1+k)k^{r-1} \quad \therefore \quad \sum_{r=1}^{\infty}OM_r=\frac{OA_1}{2}(1+k)\sum_{r=1}^{\infty}k^{r-1}$

$=\frac{OA_1}{2}(1+k)\times\frac{1}{1-k}$

$=\frac{OA_1}{2}\times\frac{1+\left(\frac{OA_2}{OA_1}\right)}{1-\left(\frac{OA_2}{OA_1}\right)}=\frac{OA_1\left(OA_1+OA_2\right)}{2\left(OA_1-OA_2\right)}$

SECTION-2

1.(8) Number of solutions of the given equation is the same as the number of solutions of the equation $x_1x_2x_3x_4=30=2\times3\times5$

Here x_4 is there because if $x_1x_2x_3=15$, then $x_4=2$ and if $x_1x_2x_3=5$, then $x_4=6$ etc.

x_4 is in fact a dummy variable.

Thus $x_1x_2x_3x_4=2\times3\times5$

Each of 2, 3, and 5 will be a factor of exactly one of x_1,x_2,x_3,x_4 in 4 ways.

$\therefore$ Required number = $4^3=64$

2.(3) We have, $f(x)=ae^{2x}+be^x+cx$,

$f(0)=-1;\ f'\left(\log 2\right)=31;\ \int_0^{\log 4}\left\{f(x)-cx\right\}dx=\frac{39}{2}$

Now, $f(0)=-1$

$\Rightarrow \quad a+b=-1 \qquad \ldots(1)$

And $f'(x)=2ae^{2x}+be^x+c$

$\Rightarrow \quad f'\left(\log 2\right)=2ae^{2\log 2}+be^{\log 2}+c=8a+2b+c$

Since, $f'(\log 2) = 31 \quad \Rightarrow \quad 8a + 2b + c = 31$

Now, $\int_0^{\log 4} \{f(x) - cx\} dx = \frac{39}{2}$

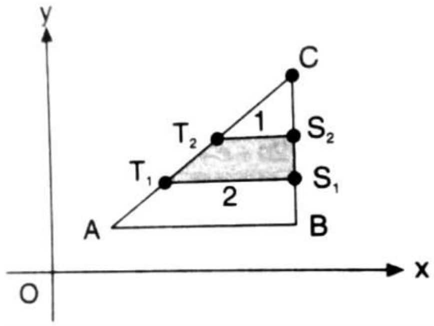
$\Rightarrow \quad \int_0^{\log 4} \{ae^{2x} + be^x\} dx = \frac{39}{2} \quad \Rightarrow \quad \left(\frac{a}{2}e^{2x} + be^x\right)_0^{\log 4} = \frac{39}{2}$

$\Rightarrow \quad \left(\frac{a}{2}e^{2\log 4} + be^{\log 4}\right) - \left(\frac{a}{2} + b\right) = \frac{39}{2} \quad \Rightarrow \quad \frac{a}{2}(16) + b(4) - \frac{a}{2} - b = \frac{39}{2}$

$\Rightarrow \quad \frac{15}{2}a + 3b = \frac{39}{2} \Rightarrow 15a + 6b = 39 \quad \Rightarrow \quad 5a + 2b = 13$

Solving (1), (2) and (3), we get $a = 5, b = -6, c = 3$

3.(3)



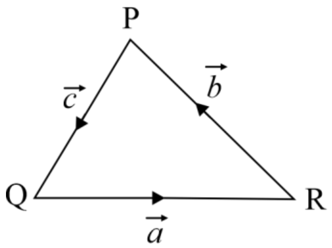
Since, AB is parallel to x-axes, and BC is parallel to y-axis,

$\therefore \quad T_1S_1S_2T_2$ form a trapezium with

$T_1S_1 = 2,\ S_1S_2 = 1$ and $T_2S_2 = 1$.

Hence, required area $= \frac{3}{2}$ sq. units.

4.(108) We have $\vec{a} + \vec{b} + \vec{c} = \vec{0}$



$\Rightarrow \quad \vec{c} = -\vec{a} - \vec{b}$

Now, $\frac{\vec{a}.(-\vec{a} - 2\vec{b})}{(-\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})} = \frac{3}{7}$

$\Rightarrow \quad \frac{9 + 2\vec{a}.\vec{b}}{9 - 16} = \frac{3}{7} \quad \Rightarrow \quad \vec{a}.\vec{b} = -6$

$\Rightarrow \quad |\vec{a} \times \vec{b}|^2 = a^2b^2 - (\vec{a}.\vec{b})^2 = 9 \times 16 - 36 = 108$

5.(1) Assuming $\cot^{-1} x = \theta$

$\Rightarrow \quad \cot\theta = x \quad \Rightarrow \quad \tan\theta = \frac{1}{x}$

Now, $e^{2i\cot^{-1}x} = e^{2i\theta} = \cos 2\theta + i\sin 2\theta$

$\Rightarrow \quad e^{2i\cot^{-1}x} = \left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) + i\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) \quad \Rightarrow \quad e^{2i\cot^{-1}x} = \left\{\dfrac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}}\right\} + i\left\{\dfrac{2\frac{1}{x}}{1+\frac{1}{x^2}}\right\}$

$\Rightarrow \quad e^{2i\cos^{-1}x} = \left(\dfrac{x^2-1}{x^2+1}\right) + i\left(\dfrac{2x}{x^2+1}\right) \quad \Rightarrow \quad e^{2i\cot^{-1}x} = \dfrac{x^2-1+2ix}{x^2+1} = \dfrac{(x+i)^2}{(x+i)(x-i)}$

$\Rightarrow \quad e^{2i\cot^{-1}x} = \dfrac{x+i}{x-i} = \dfrac{ix+i^2}{ix-i^2} = \dfrac{ix-1}{ix+1} \quad \Rightarrow \quad \left(e^{2i\cot^{-1}x}\right)^n = \left(\dfrac{ix-1}{ix+1}\right)^n$

$\therefore \quad e^{2ni\cot^{-1}x}\left(\dfrac{ix+1}{ix-1}\right)^n = 1$

6.(2) Let $I = \displaystyle\int \frac{\cos 4x + 1}{\cot x - \tan x}dx$

$\Rightarrow \quad I = \displaystyle\int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x}\sin x\cos x\,dx \quad \Rightarrow \quad I = \int \cos 2x \sin 2x\,dx = \frac{1}{4}\int \sin 4x\,dx$

$\Rightarrow \quad I = \dfrac{1}{8}\cos 4x + c \qquad \therefore \qquad A = -\dfrac{1}{8}$ and $B \in R$; So $A = 2$

7.(8) For each star, add the outer numbers together and divide by 3 for the first star, 4 for the second star, and 5 for the third star. Hence, for the fourth star, we will add all the outer numbers and divide by six.

8.(7) mean $\bar{x} = 4, \sigma^2 = 5.2, n = 5, .x_1 = 3x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9 \qquad \ldots(i)$

$\dfrac{\sum x_i^2}{x} - (\bar{x})^2 = \sigma \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65 \qquad \ldots(ii)$

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

9.(2009) Let $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$

$\therefore \quad X^1.A.X = 0$

$\Rightarrow a_{11}X_1^2 + a_{22}X_2^2 + a_{33}X_3^2 + (a_{12}+a_{21})X_1X_2 + (a_{13}+a_{31})X_1X_3 + (a_{23}+a_{32})X_2X_3 = 0$

This is true X_i

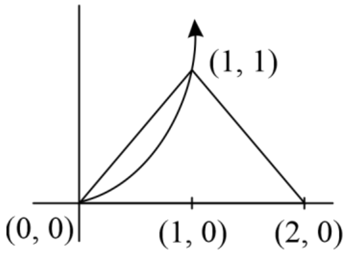
$a_{11} = a_{22} = a_{33} = 0$

$a_{12} = a_{21} = 0$

$a_{13} = a_{31} = 0$

$a_{23} = a_{32} = 0$

10.(4) Area $= \int_0^1 (x - x^n)dx = \frac{3}{10}$



$\left[\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right]_0^1 = \frac{3}{10}$

$\frac{1}{2} - \frac{1}{n+1} = \frac{3}{10}$

$\Rightarrow \quad n = 4$